# Exercise 75

For the following exercise, find a domain on which the function $f$ is one-to-one and non-decreasing. Write the domain in interval notation. Then find the inverse of $f$ restricted to that domain.

$$f(x) = x^2 + 1$$

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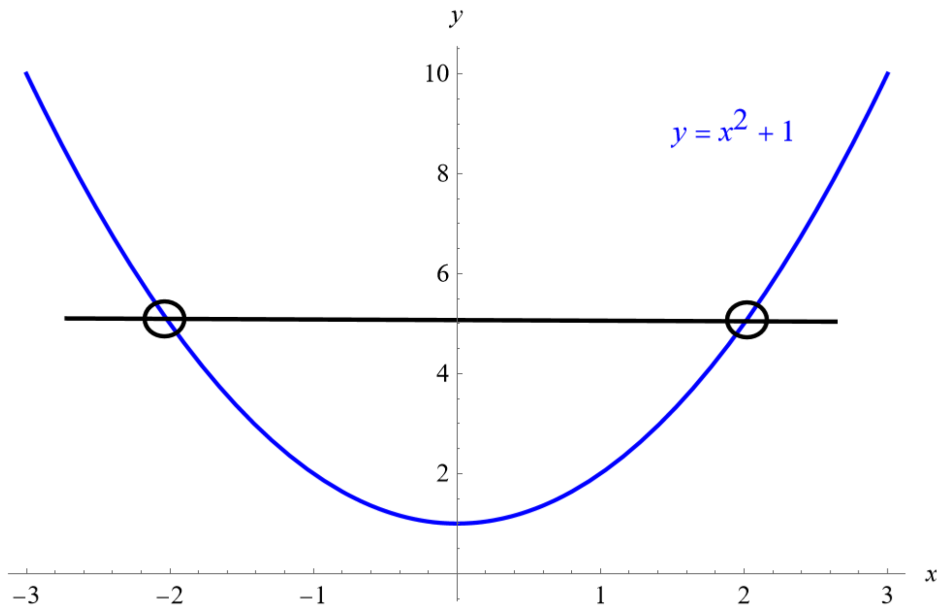
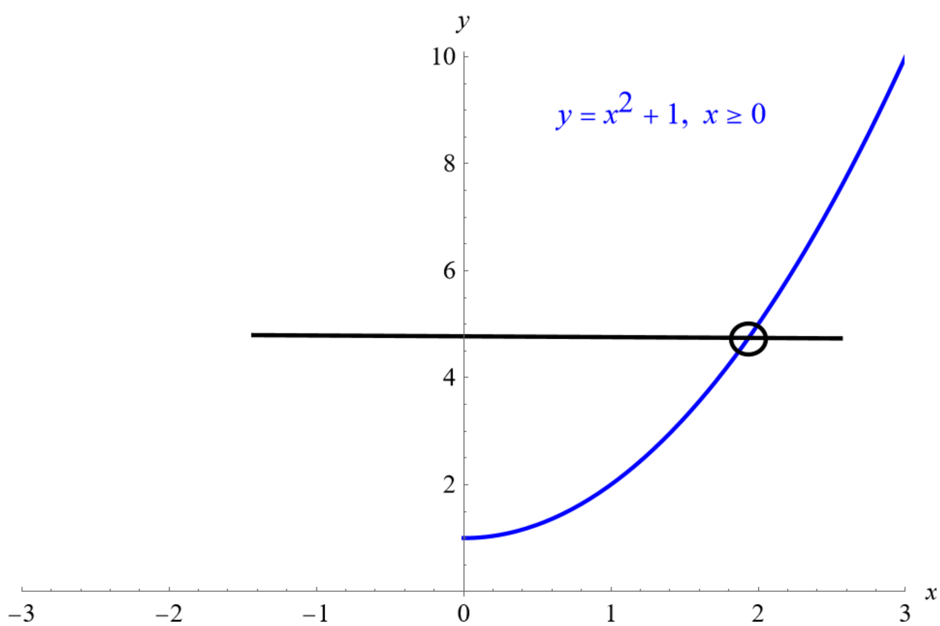
### Solution

The function as it's given is not one-to-one because it fails the horizontal line test.




However, by taking the restriction of $f(x)$ to $x \geq 0$, it can be made one-to-one.



The domain on which $f(x)$ is one-to-one and non-decreasing is $[0, \infty)$. To find the inverse function, switch $x$ and $y$ in the given equation.

$$x = y^2 + 1$$

Solve for $y$.

$$x - 1 = y^2$$

Take the square root of both sides.

$$\sqrt{x-1} = \sqrt{y^2}$$

Since there's an even power under an even root and the result is to an odd power, an absolute value sign is needed.

$$\sqrt{x-1} = |y|$$

Remove the absolute value sign by placing $\pm$ on the left side.

$$\pm\sqrt{x-1} = y$$

$y$ was originally $x$, which has a domain of $[0, \infty)$. To be consistent with this, the plus sign should be chosen. Therefore, the inverse function is

$$f^{-1}(x) = \sqrt{x-1}.$$